Appendices to

Good Principals or Good Peers? Parental Valuation of School Characteristics, Tiebout Equilibrium, and the Incentive Effects of Competition Among Jurisdictions

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Appendix A. Proofs of Propositions

Note that the budget constraint $c = x - h$ can be introduced directly into the utility function: $U(c,q) = U(x-h,q)$. The single-crossing property (eq. 1) implies that $\frac{\partial}{\partial x}\left(\frac{\partial U(x-h,q)/\partial h}{\partial U(x-h,q)/\partial q}\right) < 0$. One implication is that family *x*'s indifference curve through any given point in *q*-*h* space is steeper, the larger is *x*. This gives us the following Lemma (Dennis Epple and Richard E. Romano, 1996):

<u>Lemma 1.</u> Let $q_j > q_k$ and $h_j > h_k$. With single crossing,

i. If $U(x_0 - h_j, q_j) \geq U(x_0 - h_k, q_k)$ and $x > x_0$, $U(x - h_j, q_j) > U(x - h_k, q_k)$.

ii. If $U(x_0 - h_j, q_j) \leq U(x_0 - h_k, q_k)$ and $x < x_0$, $U(x - h_j, q_j) < U(x - h_k, q_k)$.

<u>Proof of Lemma 1.</u> I prove part i; the remainder follows directly by a similar argument. Consider *S*, the indifference curve of family x_0 through (q_k, h_k) in *q*-*h* space. By assumption, (q_j, h_j) lies on or above *S*. From single crossing, for any $x > x_0$ and any $(q,h) \in S$, *x*'s indifference curve through (q, h) is steeper than *S*. This implies that *x*'s indifference curve

through (q_k, h_k) lies strictly above *S* for all $q > q_k$, and, in particular, that it lies above *S* at $q = q_j$. The result follows directly.

Two additional results are mentioned in the text:

<u>Lemma 2.</u> There exists at least one admissible rule.

<u>Lemma 3.</u> A rule is admissible if and only if it produces perfect quality sorting.

<u>Proof of Lemma 2.</u> I prove this by construction. First, without loss of generality, let the μ_j s be sorted in descending order: $\mu_j > \mu_{j+1}$ for all $j < J$. Define $\breve{x}_j \equiv F^{-1}\left(1 - {}^{jn}\!/\!_N\right)$, the income of the *jn*-th wealthiest family, $j = 1, \ldots, J-1$. I show below that the following allocation rule is admissible:

$$\text{(A1)} \qquad \widetilde{G}(x) = \begin{cases} 1 & \text{for} \quad x \geq \breve{x}_1; \\ j & \text{for} \quad \breve{x}_{j-1} > x \geq \breve{x}_j,\ j = 2, \ldots, J; \\ J & \text{for} \quad \breve{x}_{J-1} > x. \end{cases}$$

This rule assigns the *n* highest-income families to district 1—the district with the highest μ—the next *n* families to district 2; and so on. To demonstrate admissibility, I show that this is an equilibrium with price vector $\widetilde{h} \equiv \{\widetilde{h}_1, \widetilde{h}_2, \ldots, \widetilde{h}_J\}$, where

$$\text{(A2)} \qquad \widetilde{h}_j = \begin{cases} 0 & \text{for } j = J \\ \widetilde{h}_{j+1} + \left(q_j - q_{j+1}\right) \dfrac{U_2\left(\breve{x}_j - h_{j+1},\ q_{j+1}\right)}{U_1\left(\breve{x}_j - h_{j+1},\ q_{j+1}\right)} & \text{for } j < J. \end{cases}$$

EQ1 is clear by construction of $\widetilde{G}(\cdot)$. Note that under $\widetilde{h}$, family $\breve{x}_j$ is indifferent between *j* and *j*+1, so EQ2 comes directly from Lemma 1. By definition of $\widetilde{G}(\cdot)$, $\bar{x}_j > \bar{x}_k$ whenever $\mu_j > \mu_k$, so $q_j = \bar{x}_j \delta + \mu_j > \bar{x}_k \delta + \mu_k = q_k$ for any $\delta \geq 0$. In particular, $q_j \neq q_k$ and EQ3 is satisfied.

Proof of Lemma 3. Suppose that G is an allocation rule that does not produce perfect quality sorting: For some w and some $y>w$ with $G(y) \neq G(w)$, $q_{G(y)} \leq q_{G(w)}$. Suppose that this is an equilibrium with price vector h. By EQ2, household w must prefer community $G(w)$ to community $G(y)$ with these housing prices. But since $y>w$ and $q_{G(y)} \leq q_{G(w)}$, Lemma 1 implies that y also prefers $G(w)$ to $G(y)$, violating EQ2.

This brings us to the propositions stated in the text.

Proof of Proposition 1. Let $r_G(j)$ be the mapping from community index numbers to quality rank in some allocation rule G, so $q_j > q_k \Leftrightarrow r_G(j) < r_G(k)$. The Proposition states that the following are necessary conditions for G to be an equilibrium allocation with housing price vector ($h_1, \ldots, h_J$):

i. $h_j > h_k$ whenever $r_G(j) < r_G(k)$,

ii. if $x \geq \breve{x}_1$, then $r_G(G(x)) = 1$, and

iii. if $\breve{x}_{j-1} > x \geq \breve{x}_j$ for $j = 2, \ldots, J$, then $r_G(G(x)) = j$.

I prove this by contradiction. (i) is easily dismissed: If $r_G(j) < r_G(k)$ but $h_j \leq h_k$, then j dominates k in every family's preferences. To satisfy EQ2, $G^{-1}(k)$ must be the empty set. But a community cannot be empty, by EQ1 and $N > n(J-1)$, so this is impossible.

Now assume that (i) holds, but (ii) or (iii) does not. There must be some community j, with rank $r = r_G(j) < J$, and some $x_0 \geq \breve{x}_r$, such that $r_G(G(x_0)) > r$. This, in turn, requires that there be some *k*, with $r_G(k) \le r$, such that either $G(x_1) = k$ for some $x_1 < \breve{x}_r$ or $\int 1(G(x) = k)dF(x) < {}^{n}\!/\!_{N}$. The first possibility violates EQ2, by Lemma 1. The second, with EQ1, implies $h_k = 0$; in this case, no community with quality less than q_k (of which there must be at least one, since $r_G(k) \le r < J$) attracts any residents with nonnegative prices. Again, there are not enough communities for one to be empty, so this is impossible.

<u>Proof of Proposition 2.</u> By Proposition 1, equilibrium allocations amount to permutations of *J* bins of the income distribution among the *J* communities, with the restriction that wealthier families live in higher-quality districts. When $\delta = 0$, $q_j \equiv \bar{x}_j\delta + \mu_j \equiv \mu_j$, so the only possible quality ranking is the ranking by effectiveness and only one permutation is admissible.

To demonstrate the second portion of the proposition, let *G* be an admissible rule with preferences δ_0 and, without loss of generality, assume $q_1 > q_2 > \ldots > q_J$ when preferences are described by δ_0. By Proposition 1, *G* must assign the *n* wealthiest households to community 1; the next *n* to 2, and so on. Letting $\breve{x}_j \equiv F^{-1}\left(1 - {}^{jn}\!/\!_{N}\right)$, for any δ we can define housing prices

$$\text{(A3)} \qquad \tilde{h}_j(\delta) = \begin{cases} 0 & \text{for } j = J \\ \tilde{h}_{j+1}(\delta) + \left(q_j(\delta) - q_{j+1}(\delta)\right)\dfrac{U_2\left(\breve{x}_j - h_{j+1},\, q_{j+1}(\delta)\right)}{U_1\left(\breve{x}_j - h_{j+1},\, q_{j+1}(\delta)\right)} & \text{for } j < J, \end{cases}$$

where the notation $q(\delta)$ and $h(\delta)$ indicates that both perceived quality and prices are functions of δ. These prices satisfy EQ1-EQ3 whenever $\delta > \delta_0$. EQ1 is a property of *G*, so is invariant to δ. Note that $q_j(\delta) - q_{j+1}(\delta) = (\bar{x}_j - \bar{x}_{j+1})\delta + \mu_j - \mu_{j+1} > (\bar{x}_j - \bar{x}_{j+1})\delta_0 + \mu_j - \mu_{j+1} = q_j(\delta_0) - q_{j+1}(\delta_0) > 0$, where $\bar{x}_j$ is the average income of households assigned to community *j* by *G* (and therefore

$\bar{x}_j > \bar{x}_{j+1}$), so EQ1 is satisfied and the quality ranking is unchanged. By construction of $\tilde{h}(\delta)$, household $\breve{x}_j$ is indifferent between *j* and *j*+1 with preferences *δ*, so Lemma 1 implies EQ2.

Appendix B. Choice and Stratification

This Appendix presents evidence that the district choice index, c_m, captures meaningful variation in parents' ability to exercise Tiebout choice. This evidence derives from tests of two predictions. First, we might expect that MSAs offering more public-sector choice would have lower rates of private school enrollment. Second, the most basic implication of Tiebout-style models like that in Section I is that high-choice markets should exhibit more stratification across schools (Randall W. Eberts and Timothy J. Gronberg, 1981; Epple and Holger Sieg, 1999).

Bivariate correlations in Table 1 support both hypotheses: Choice is negatively correlated with private enrollment (ρ=-0.10) and positively correlated with racial segregation across schools (ρ=0.36 or 0.22, depending on the segregation measure used).[1] Appendix Table 1 presents regression results. Columns 1 and 2 present models for metropolitan private enrollment rates, first with a vector of conventional demographic controls (the same as those used in *Z* in the primary specification in the main text), and second adding measures of census-tract-level residential segregation in the MSA. The regressions indicate that the district-choice index is a strong predictor of the private enrollment rate, suggesting that MSAs with high index values impose fewer constraints on parents' ability to choose within the public sector.

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[1] Income stratification would be preferable to racial segregation, but the data on school racial composition—here, from censuses of public (the Common Core of Data) and private (the Private School Survey) schools—are much better than those on family income.

The remaining columns of Appendix Table 1 present estimates of the relationship between the choice index and measures of student racial stratification across schools. Using both isolation and dissimilarity indices (David M. Cutler et al., 1999) for the distribution of white students relative to nonwhites and measuring these indices over either public or public and private schools, Columns 3 through 6 indicate, again, that the choice index has a large effect in the expected direction.

References

Cutler, David M.; Glaeser, Edward L. and Vigdor, Jacob L. "The Rise and Decline of the American Ghetto." *Journal of Political Economy*, 1999, *107*(3), pp. 455-506.

Eberts, Randall W. and Gronberg, Timothy J. "Jurisdictional Homogeneity and the Tiebout Hypothesis." *Journal of Urban Economics*, 1981, *10*(2), pp. 227-39.

Epple, Dennis and Romano, Richard E. "Ends against the Middle: Determining Public Service Provision When There Are Private Alternatives." *Journal of Public Economics*, 1996, *62*(3), pp. 297-325.

Epple, Dennis and Sieg, Holger. "Estimating Equilibrium Models of Local Jurisdictions." *Journal of Political Economy*, 1999, *107*(4), pp. 645-81.

Appendix Table 1: Choice as a predictor of private enrollment rates and of the racial segregation of schools.

	MA Private Enrollment Rate		Measures of White/Non-White Segregation			
			Dissimilarity		Isolation	
			All HS	Public HS	All HS	Public HS
	(A)	(B)	(C)	(D)	(E)	(F)
Choice index	-0.05	-0.05	0.13	0.15	0.12	0.16
	(0.01)	(0.01)	(0.03)	(0.04)	(0.04)	(0.05)
ln(Population) / 100	0.99	0.75	-0.42	-0.52	-2.99	-3.89
	(0.29)	(0.28)	(0.83)	(0.93)	(2.41)	(2.72)
Pop.: Fr. Black	0.12	0.07	0.16	0.08	0.24	0.06
	(0.05)	(0.07)	(0.11)	(0.13)	(0.18)	(0.20)
Pop.: Fr. Hispanic	-0.05	-0.05	0.05	0.06	0.00	-0.04
	(0.03)	(0.03)	(0.04)	(0.05)	(0.07)	(0.07)
Mean log HH income	0.03	0.02	0.01	0.00	0.17	0.18
	(0.03)	(0.03)	(0.04)	(0.05)	(0.09)	(0.10)
Gini, HH income	0.29	0.16	0.05	-0.13	0.64	0.45
	(0.17)	(0.16)	(0.27)	(0.31)	(0.50)	(0.56)
Pop: Fr. BA+	0.04	0.12	0.05	0.13	-0.21	-0.21
	(0.06)	(0.07)	(0.16)	(0.17)	(0.32)	(0.35)
Finance: Fndtn. Plan	0.02	0.02	-0.01	-0.01	-0.06	-0.06
	(0.01)	(0.01)	(0.02)	(0.02)	(0.05)	(0.06)
Finance: Slope /100	0.16	0.17	0.23	0.15	-0.30	-0.64
	(0.19)	(0.17)	(0.38)	(0.44)	(0.67)	(0.74)
Tract-level segregation measures						
Dissimilarity Index		0.03	1.06	1.11	0.25	0.26
		(0.07)	(0.13)	(0.14)	(0.24)	(0.27)
Isolation Index		0.06	-0.25	-0.20	0.61	0.72
		(0.08)	(0.11)	(0.13)	(0.15)	(0.17)
X-tract share of variance, log(HH inc.)		-0.06	0.33	0.33	0.37	0.45
		(0.09)	(0.16)	(0.19)	(0.33)	(0.38)
R^2	0.52	0.53	0.84	0.82	0.70	0.67

Notes: Observations are MSAs; N=318 (287 in Columns C-F, which exclude MSAs missing racial composition for schools with more than 25 percent of enrollment). All models include fixed effects for 8 census divisions. All standard errors are clustered on the (C)MSA.